Indian Statistical Institute
Mid-Semestral Examination
Differential Geometry I
MMath I

Max Marks: 40   Time: 3 hours

(1) (a) Define the terms: curvature, signed curvature. How are the signed curvature and the curvature of a unit-speed plane curve related? [3]

(b) Let σ be a unit-speed plane curve. Show that $\dot{\mathbf{n}}_s = -\kappa_s \mathbf{t}$ where $\mathbf{n}_s$, κ_s and $\mathbf{t}$ denote the signed unit normal, the signed curvature and the tangent vector respectively. Show that the signed curvature κ_s is smooth. [4]

(c) A unit-speed plane curve σ has the property that its tangent vector $\mathbf{t}(s)$ makes a fixed angle θ with $\sigma(s)$ for all s. Show the following: (i) If $\theta = 0$, then σ is a part of a straight line, (ii) If $\theta = \pi/2$, then σ is part of a circle. [7]

(2) (a) Define the terms: principal normal, binormal and torsion. [3]

(b) Let σ be a regular curve in $\mathbb{R}^3$ with nowhere vanishing curvature. Show that the image of σ is contained in a plane if and only if its torsion is zero at every point. [6]

(c) Let σ be a unit-speed curve in $\mathbb{R}^3$ whose principal normal $\mathbf{n}$ always points towards the origin. In other words, there is a function $\lambda(s) > 0$ such that $\sigma(s) + \lambda(s)\mathbf{n}(s) = 0$ for all s. Find the curvature and torsion of σ. What is the image of σ? [5]

(3) (a) Define the terms: smooth surface, orientable surface. [4]

(b) Show that the Mobius band is a non-orientable smooth surface. [8]